# UK INTERMEDIATE MATHEMATICAL CHALLENGE

## THURSDAY 7TH FEBRUARY 2013

Organised by the **United Kingdom Mathematics Trust**
and supported by

**The Actuarial Profession**
making financial sense of the future

**RULES AND GUIDELINES** (to be read before starting)

1.  Do not open the paper until the Invigilator tells you to do so.

2.  Time allowed: **1 hour**.
    No answers, or personal details, may be entered after the allowed hour is over.

3.  The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.

4.  Candidates in England and Wales must be in School Year 11 or below.
    Candidates in Scotland must be in S4 or below.
    Candidates in Northern Ireland must be in School Year 12 or below.

5.  **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the
    Answer Sheet for each question. Do not mark more than one option.

6.  *Do not expect to finish the whole paper in 1 hour.* Concentrate first on Questions 1-15.
    When you have checked your answers to these, have a go at some of the later questions.

7.  Five marks are awarded for each correct answer to Questions 1-15.
    Six marks are awarded for each correct answer to Questions 16-25.
    **Each incorrect answer to Questions 16-20 loses 1 mark.**
    **Each incorrect answer to Questions 21-25 loses 2 marks.**

8.  Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the
    sheet except to mark your chosen options**. The machine 'sees' all black pencil markings
    even if they are in the wrong places.  If you mark the sheet in the wrong place, or leave bits of
    rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.

9.  The questions on this paper challenge you to **think**, not to guess. You get more marks,
    and more satisfaction, by doing one question carefully than by guessing lots of answers.
    The UK IMC is about solving interesting problems, not about lucky guessing.

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1. Which of the following is divisible by 6?

   A  one million minus one     B  one million minus two     C  one million minus three
        D  one million minus four     E  one million minus five

2. A machine cracks open 180 000 eggs per hour. How many eggs is that per second?

   A  5      B  50      C  500      D  5000      E  50 000

3. How many quadrilaterals are there in this diagram, which is constructed using 6 straight lines ?

   A  4      B  5      C  7      D  8      E  9
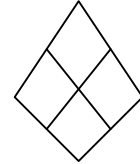
4. A standard pack of pumpkin seeds contains 40 seeds. A special pack contains 25% more seeds. Rachel  bought a special pack and 70% of the seeds germinated. How many pumpkin plants did Rachel  have?

   A  20      B  25      C  28      D  35      E  50

5. The northern wheatear is a small bird weighing less than an ounce. Some northern wheatears migrate from sub-Saharan Africa to their Arctic breeding grounds, travelling almost 15 000 km. The journey takes just over 7 weeks. Roughly how far do they travel each day, on average?
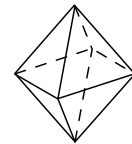
   A  1 km      B  9 km      C  30 km      D  90 km      E  300 km

6. Which of the following has the least value?

   A  $1^0 - 0^1$      B  $2^1 - 1^2$      C  $3^2 - 2^3$      D  $4^3 - 3^4$      E  $5^4 - 4^5$

7. The faces of a regular octahedron are to be painted so that no two faces which have an edge in common are painted in the same colour. What is the smallest number of colours required?

   A  2      B  3      C  4      D  6      E  8

8. Jim rolled some dice and was surprised that the sum of the scores on the dice was equal to the product of the scores on the dice. One of the dice showed a score of 2, one showed 3 and one showed 5. The rest showed a score of 1. How many dice did Jim roll?

   A  10      B  13      C  17      D  23      E  30

9. Jane has 20 identical cards in the shape of an isosceles right-angled triangle. She uses the cards to make the five shapes below. Which of the shapes has the shortest perimeter?
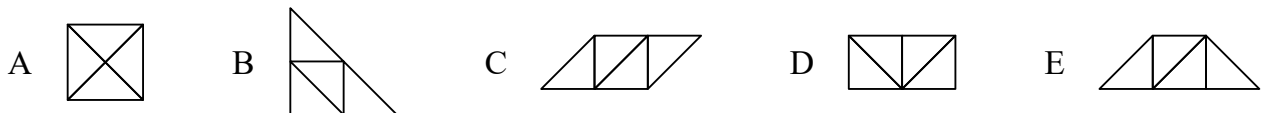
   A     B     C     D     E

10. *ABCDE* is a regular pentagon and *BCF* is an equilateral triangle such that *F* is inside *ABCDE*. What is the size of ∠*FAB*?

    A  48°      B  63°      C  66°      D  69°      E  72°

11. For which of the following numbers is the sum of all its factors *not* equal to a square number?

    A  3      B  22      C  40      D  66      E  70

12. The sum                 one + four = seventy

    becomes correct if we replace each word by the number of letters in it to give $3 + 4 = 7$.
    Using the same convention, which of these words could be substituted for $x$ to make the sum
    three + five = $x$   true?

    A  eight          B  nine          C  twelve          D  seventeen          E  eighteen

13. Four congruent isosceles trapeziums are placed so that their longer
    parallel sides form the diagonals of a square $PQRS$, as shown. The point
    $X$ divides $PQ$ in the ratio 3:1. What fraction of the square is shaded?
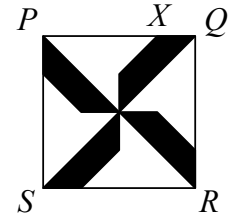
    A  $\dfrac{5}{16}$      B  $\dfrac{3}{8}$      C  $\dfrac{7}{16}$      D  $\dfrac{5}{12}$      E  $\dfrac{1}{2}$

14. Which of the following has the greatest value?

    A  $\left(\dfrac{11}{7}\right)^3$          B  $\left(\dfrac{5}{3}\right)^3$          C  $\left(\dfrac{7}{4}\right)^3$          D  $\left(\dfrac{9}{5}\right)^3$          E  $\left(\dfrac{3}{2}\right)^3$

15. I have a bag of coins. In it, one third of the coins are gold, one fifth of them are silver, two
    sevenths are bronze and the rest are copper. My bag can hold a maximum of 200 coins. How
    many coins are in my bag?

    A  101      B  105      C  153      D  195      E  more information is needed

16. Which diagram shows the graph of $y = x$ after it has been rotated 90° clockwise about the
    point (1, 1)?

          A                   B                   C                   D                   E
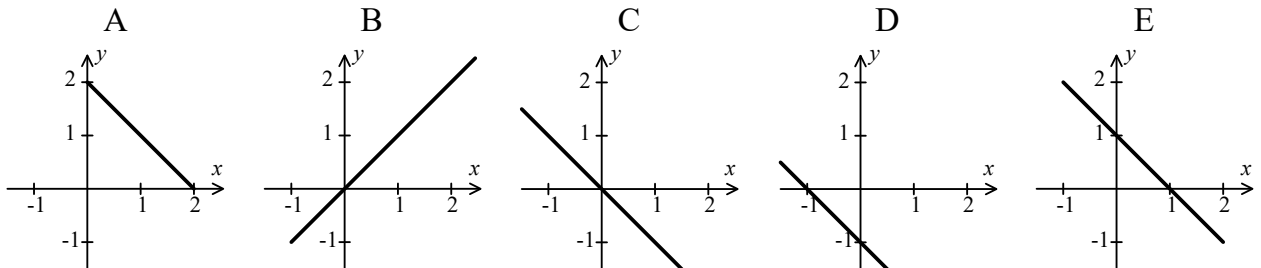
17. The diagram shows four equal discs and a square. Each disc
    touches its two neighbouring discs. Each corner of the square is
    positioned at the centre of a disc. The side length of the square is
    $2 / \pi$. What is the length of the perimeter of the figure?
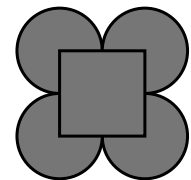
    A  3      B  4      C  $\dfrac{3\pi}{2}$      D  6      E  $2\pi$

18. The triangle $T$ has sides of length 6, 5, 5. The triangle $U$ has sides of length 8, 5, 5.

    What is the ratio area $T$ : area $U$?

    A  9 : 16      B  3 : 4      C  1 : 1      D  4 : 3      E  16 : 9

19. Which of the expressions below is equivalent to $(x \div (y \div z)) \div ((x \div y) \div z)$ ?

    A  1      B  $\dfrac{1}{xyz}$      C  $x^2$      D  $y^2$      E  $z^2$

20. Jack's teacher asked him to draw a triangle of area 7cm$^2$. Two sides are to be of length 6cm and 8cm. How many possibilities are there for the length of the third side of the triangle?

   A  1              B  2              C  3              D  4              E  more than 4

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21. The square *ABCD* has an area of 196. It contains two overlapping squares; the larger of these squares has an area 4 times that of the smaller and the area of their overlap is 1.  What is the total area of the shaded regions?

   A  44            B  72            C  80            D  152
   E  more information is needed
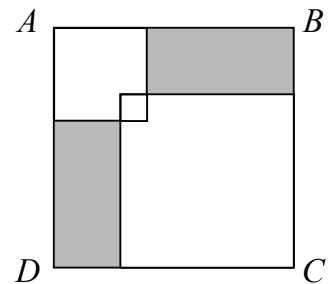


22. The diagrams show squares placed inside two identical semicircles. In the lower diagram the two squares are identical.
   What is the ratio of the areas of the two shaded regions?
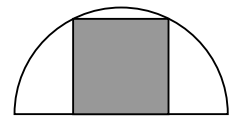
   A  1 : 2   B  2 : 3   C  3 : 4   D  4 : 5   E  5 : 6
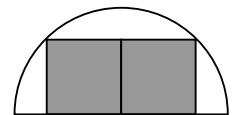


23. Four brothers are discussing the order in which they were born. Two are lying and two are telling the truth. Which two are telling the truth?

   Alfred:  "Bernard is the youngest."    Horatio: "Bernard is the oldest and I am the youngest."
   Inigo: "I was born last."                Bernard:  "I'm neither the youngest nor the oldest."

   A  Bernard and Inigo              B  Horatio and Bernard            C  Alfred and Horatio
               D  Alfred and Bernard                  E  Inigo and Horatio

24. The diagram shows a shaded shape bounded by circular arcs with the same radius. The centres of three arcs are the vertices of an equilateral triangle; the other three centres are the midpoints of the sides of the triangle. The sides of the triangle have length 2.
   What is the difference between the area of the shaded shape and the area of the triangle?
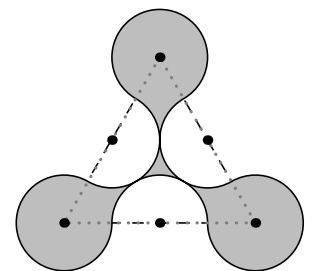


   A  $\dfrac{\pi}{6}$        B  $\dfrac{\pi}{4}$        C  $\dfrac{\pi}{3}$        D  $\dfrac{\pi}{2}$        E  $\pi$

25. In 1984 the engineer and prolific prime-finder Harvey Dubner found the biggest known prime each of whose digits is either a one or a zero. The prime can be expressed as
   $\dfrac{10^{641} \times (10^{640} - 1)}{9} + 1$.  How many digits does this prime have?

   A  640              B  641              C  1280              D  1281              E  640 × 641